Multi-Disciplinary Cyber-Physical Optimization for Unmanned Aircraft Systems

Justin M. Bradley* and Ella M. Atkins†
University of Michigan, Ann Arbor, MI, USA

Future small unmanned aircraft systems (UAS) will require careful co-design over both physical and cyber elements to maximize total-system efficiency. Mission objectives and success of the system as a whole are becoming increasingly dependent on appropriate allocation of computational resources balanced against demands of the physical actuation systems. In this paper we describe a co-optimization scheme considering tradeoffs between costs associated with physical actuation effort required for control and computational effort required to acquire and process incoming information. A small UAS surveillance mission, visual inspection of a pipeline, is proposed to investigate specifics of cyber-physical cost terms and their tradeoffs. A multi-disciplinary cost function minimizes energy and maximize mission efficiency and effectiveness. We examine Pareto fronts over combinations of competing cyber and physical objectives and demonstrate that excluding either cyber or physical cost terms results in reduced performance for the holistic system over the course of the mission.

Nomenclature

- $\alpha$: Tuning parameter in entropy cost
- $\delta_t$: Throttle setting
- $\eta$: Propeller efficiency factor
- $\rho$: Air density
- $\rho_s$: Air density at sea level
- $\tau$: Real-time system task achieving important mission goal
- $A$: Area of an image
- $b$: Wingspan
- $C_{D_0}$: Zero-lift drag coefficient
- $D$: Total distance of mission
- $E$: Energy for the physical system
- $e_0$: Oswald efficiency factor
- $f$: Focal length of camera
- $H$: Entropy cost
- $H_{dist}$: Horizontal distance of imaging plane
- $J_p$: Cost function for physical system
- $K$: Aerodynamic parameter
- $L$: Lagrangian of constrained cost function
- $m$: Mass of aircraft
- $P(v)$: Power as a function of velocity
- $P_{max}$: Maximum power of the engine/motor at sea level
- $r_\tau$: Task execution rate in Hz of task $\tau$
- $S$: Surface area of wing
- $S_e$: Wing surface area

*PhD Student, Aerospace Engineering Dept., AIAA Student Member
†Associate Professor, Aerospace Engineering Dept., AIAA Associate Fellow
The primary goal of air vehicle design and operation has historically been to achieve appropriate maneuverability and to overcome aerodynamic drag and the influence of gravity in a manner that maximizes range and/or endurance. To-date, the energy a powered aircraft requires to apply the necessary propulsive and control actuation forces over a flight has dominated the total energy consumed across all other vehicle subsystems. Surveillance Unmanned Aircraft Systems (UAS) are becoming smaller and are constructed with composite materials that minimize weight. They are also being equipped with increasingly sophisticated avionics and payloads. Powered glider designs in particular exhibit low drag and weight, resulting in a significantly reduced thrust requirement. For the first time, the power required by avionics and payload systems for a flight vehicle is comparable to propulsive plus control actuation requirements. We will likely see a future where avionics and payload power can even exceed power required for force application particularly during periods of demanding on-board processing and communication activity.

Control systems engineers typically optimize vehicle trajectories and thereby force application histories over physics-based models of vehicle dynamics, including flight envelope and actuator saturation constraints. Conversely, software engineers optimize processor and communication resource use over the suite of computational and information sharing tasks, regulating energy use through the real-time regulation of variable-speed processors, activation/shutdown of cores in a multi-processor system, and regulation of communication links. While real-time task execution models are typically discrete rather than continuous-time, the methods used to optimally control physical and real-time computing systems are fundamentally the same: gradient or search-based algorithms are used to identify minimum-cost solutions given constraints.

For emerging UAS that consume comparable power for avionics versus force application, neither physical nor cyber system optimization is dominant. Therefore, globally-optimal (minimum-energy, minimum-time, maximum-information) performance can only be achieved if cyber and physical models can be shown independent of each other, or else if necessary cyber and physical couplings are identified and simultaneously considered during optimization. Computational resources must be utilized as a minimum to guide, navigate, and control the UAS as well as to compute or update future spatiotemporal (4-D) trajectories. Physical trajectories in turn enable the cyber system to maximize its ability to acquire information (e.g., from sensors) and to communicate (e.g., to ground operators). Cyber and physical resources are therefore necessarily coupled.

This paper presents a new multi-disciplinary optimization direction for which the models being integrated optimize energy consumption over both physical effectors and computational (cyber) resources. The cost function to be optimized includes weighted terms representing energy used by physical actuators and energy used by a multi-core or variable-speed processing architecture. This paper presents a case study and appropriate UAS mission aimed at assessing the potential performance improvements possible with co-optimization of cyber and physical resources over energy use, time, and mission accomplishment. A mission-appropriate analytical cost function is developed to provide a minimal-cost trajectory over the mission. We simplify the cost function by allowing design variables to remain static throughout the mission, consistent with a steady flight scenario, thereby reducing the complexity of the cost function and optimization process. We then examine Pareto fronts for combinations of cost function objectives to demonstrate the important tradeoffs between physical and cyber resources and to give insight into the interdependence between them. We use a numerical solver to find physical-subsystem optimal, cyber-subsystem optimal, and holistic system optimal solutions and compare them with solutions selected from Pareto front analysis. We demonstrate that only via a total cyber-physical system (CPS) optimization can one achieve efficiency throughout the total system.

For our case study, we adopt a solar-supplemented powered-glider small UAS currently flown by a University of Michigan student team (SolarBubbles) for which steady flight performance parameters are available.
The small UAS payload is a downward-facing video camera that can provide frames at a variable rate. The simulated avionics allows direct regulation of computational power requirements (energy use) in a manner that trades energy use with camera data acquisition bandwidth. A one-dimensional pipeline inspection case study is investigated, focusing attention on physical and computational energy use tradeoffs without complexity in the actual path through three-dimensional space.

II. Cost Functions

We seek optimization over both physical and cyber characteristics of the UAS in order to more holistically optimize system performance for a designated mission. This means developing cost terms for task performance and energy required for cyber activities as well as for control actuation effort and propulsion. Moreover, we naturally want to maximize efficiency of our designated mission which will include goals for both the physical and cyber components of the UAS. These mission-dependent goals may include maximizing coverage area or amount of information acquired for a given area, along with collection, processing and transmission of data.

In this work we develop cost terms representing both mission goals and efficiency for a UAS whose mission is surveillance of a straight section of pipeline with a small, lightweight downward facing gimbaled camera. Because of this we focus on movement in one dimension only and assume flight dynamics are governed by steady flight assumptions.

Our objective for the physical system will be to determine the optimal velocity (airspeed in one dimension) of the UAS for the mission. Owing to the assumptions of the mission and steady flight we rely on a gimbal to consistently adjust the camera to point directly toward the ground (optical axis perpendicular to ground plane) which compensates for changes in pitch of the aircraft needed to accommodate various velocities of flight.

We model a single real-time task to accomplish the primary goals of the mission related to pipeline surveillance. This task performs image acquisition, processing, and communication/storage of image. Our design objective for the cyber system will be to determine the optimal execution rate of this task. While there are other system-critical tasks on the cyber system including the control task, we assume these require a fixed amount of resources. We instead focus on optimizing over the remaining non-critical bandwidth available in the cyber system.

We divide the cost terms into physical and cyber goals for clarity, and to emphasize the assimilation of each into a system-wide cost function. “Physical” in the context of a UAS includes items related to flight, for example, the airframe, propulsion system, and control surfaces. “Cyber” relates to items required for data processing, communication, image collection, computation of control inputs, etc. In this work we endeavor to focus clearly on the idea of combining physical and cyber cost terms into a holistic cyber-physical system cost function.

A. Physical System Terms

Small UAS often have very modest energy reserves, most often consisting of small battery packs, or a small fuel tank. In non-energy-harvesting applications under normal conditions such energy supplies can provide between thirty minutes to multiple hours of flight time. These flight times can be reduced when cyber-intensive activities such as image processing and communication are involved. Minimizing energy consumption during steady flight is an important consideration in the design and control of the UAS.

1. Physical System Energy

In most aircraft applications, propulsion will consume the majority of the energy required for flight, surpassing actuation effort required by control surfaces. For simplicity, in this work we assume propulsion is the only drain on energy supplies by the physical system and we model steady flight in which power used by control surface servos would be constant or near-constant. We therefore seek to minimize energy of the physical system over the entire mission

\[ E = \int P(v(t)) \, dt \]
where $P(v(t))$ is a traditional model for power as a function of velocity\(^3\)

$$P(v) = \frac{1}{2} C_D \rho v^3 + \frac{2KW^2}{\rho v(t) \rho_{air}S}.$$  \hspace{1cm} (2)

In steady level flight, power of the aircraft, and therefore velocity, is manipulated by a throttle setting that maps nonlinearly to power as

$$P = \eta \delta t \left( \frac{\rho}{\rho_s} \right)^m P_{\text{max}}^s$$  \hspace{1cm} (3)

where $P_{\text{max}}^s$ is the maximum power of the engine/motor at sea level, $m > 0$ is a characteristic of the engine/motor, $0 \leq \eta \leq 1$ is a propeller efficiency factor, $\rho_s$ is the air density at sea level, and $0 \leq \delta t \leq 1$ is the throttle setting. The power curve for our UAS (described in Section 1) can be seen in Figure 1.

2. Time

In addition to minimizing energy, ideally we would like to efficiently accomplish our mission by minimizing the time required to complete it. Such time-minimal optimization cost terms appear frequently and are simply given by

$$T = \int dt.$$  \hspace{1cm} (4)

3. Cost Function for Physical System

These two competing objectives, $E_p$ and $T$, comprise the cost function for the overall physical system

$$J_p(v(t)) = \beta_{p1} \int P(v(t)) \, dt + \beta_{p2} \int dt$$  \hspace{1cm} (5)

where $\beta_{p1}, \beta_{p2}$ are weighting terms. Optimizing $J_p$ alone is what a traditional trajectory or path planner would do if no costs are attributed to the cyber system. While some UAS researchers have added tracking information, target acquisition, and other mission objectives to their control and optimization algorithms, to our knowledge this has historically been done from the physical perspective without attempting to optimize over cyber system performance and requirements.

B. Cyber System Terms

In a modern fully-autonomous UAS the cyber system becomes the gateway for virtually all aspects of the system. Control actuation inputs, data collection, communication, throttle setting, path and mission planning are potentially all being done simultaneously on-board. While real-time system researchers have advanced scheduling techniques for prioritizing each of these critical tasks, the correlation between physical performance, mission objectives, and computational efficiency has remained largely unexplored.

In many cyber-physical systems (CPS) task execution rates are selected \textit{a priori} based on requirements of the system. For example, the sampling rate of the control task may be selected based on digital control analysis. While it is unreasonable to interfere with such high priority tasks, lower priority tasks may still have some flexibility in task execution rate allowing us to optimize over mission and cyber parameters without interfering with mission critical tasks.

In our previous work we explored the tradeoff of mission critical task execution rates and physical performance.\(^6,7\) For this work we assume that hard real-time feedback control tasks are appropriately executed while we focus on the rest of the available cyber resources for soft real-time tasks. More specifically, we
assume that we cannot only conserve energy by optimally selecting execution rates of lower priority tasks, but we can also increase mission effectiveness by developing costs that relate task execution rates to mission efficiency.

1. Cyber Utilization

We assume that at least part of the cyber utilization is fixed based on selected and scheduled rates of mission critical tasks, and instead focus on maximizing use of the remaining resources. To that end, we assume a single task, \( \tau \), achieves the important mission goal of capturing and processing an image of the pipeline. The task runs at execution rate \( r_\tau (k) \) Hz and has a maximum execution rate of \( r_{\tau, \text{max}} \) Hz stemming from restrictions based on available cyber resources. We introduce the cyber utilization term

\[
U_\tau = \sum_k \frac{r_\tau (k)}{r_{\tau, \text{max}}}. \tag{6}
\]

We note that \( r_\tau (k) \) is the rate of execution of task \( \tau \) at discrete intervals \( k \) and that the rate of execution cannot change during a particular execution cycle of that task. We assume that cyber utilization is proportional to energy consumed by the cyber system, and as a result, minimizing it is the cyber equivalent to the energy minimization term of the physical system in Equation (1).

2. Mission Information

We seek to relate mission efficiency to cyber and physical parameters. For our specified mission, we assume that detailed imagery of the pipeline is critical for detecting aberrations and problems. Therefore, we create a cost term based on overlap between successive images where increasing overlap is rewarded. Acquiring multiple images of the same ground points has the advantage of providing additional viewpoints and redundant data, and may allow for super-resolved imagery thereby increasing our ability to detect pipeline anomalies.\(^8\)

From an information theory perspective we view information cost as entropy for which an exponential distribution indicates the quantity of unknown information associated with points on the overflown region (or pipeline). In this sense, minimizing the entropy has the effect of maximizing the total information acquired. This term has the effect of requiring a combination of slow aircraft speed and/or increased task frequency and depends on both velocity of the aircraft, \( v(t) \), and rate of image acquisition and processing \( r_\tau (k) \)

\[
H = \int e^{-\alpha \Omega(v(t), r_\tau (k))} dt. \tag{7}
\]

We let \( \alpha \) be a tuning parameter, and

\[
\Omega(v(t), r_\tau (k)) = \frac{1}{A} \left( A - w \int_{t-T_\tau (k)}^t v(\gamma) d\gamma \right) \tag{8}
\]

where \( A \) is the total area of an image, \( w \) is the width of an image, and \( T_\tau (k) = \frac{1}{r_\tau (k)} \) is the period of task \( \tau \). For simplicity, we assume that the aircraft flies at approximately the same height above ground for the mission, and therefore \( A \) and \( w \) remain constant. In Figure 2 we show a plot of \( H \) to demonstrate how the entropy changes with both cyber rate \( (r_\tau) \) and velocity \( (v) \). The dependence of entropy on cyber rate falls off as a steep exponential, and falls off more gradually with aircraft velocity. This means we expect our Pareto front analysis in Section A to indicate lower entropy by increasing cyber rate than by going slower.
3. Cost Function for Cyber System

The expressions in Equations (6) and (7) comprise the cost function for the cyber system

\[
J_c(v(t), r_\tau(k)) = \beta_{c1} \frac{r_\tau(k)}{r_{\tau,\text{max}}} + \beta_{c2} \int e^{-\alpha \Omega(v(t), r_\tau(k))} dt \tag{9}
\]

where we have weighting terms \( \beta_{c1} \) and \( \beta_{c2} \). Such a cost function might be used if we were only interested in trading cyber resource utilization cost against reward from accomplishing mission objectives, which in the pipeline inspection case study maps to minimizing entropy (unknown information) that could be obtained through overlapping image data acquisition and processing.

C. CPS Cost Function

We combine \( J_p \) and \( J_c \) to obtain a holistic CPS cost function

\[
J(v(t), r_\tau(k)) = \beta_{p1} \int P(v(t)) dt + \beta_{p2} \int dt + \beta_{c1} \frac{r_\tau(k)}{r_{\tau,\text{max}}} + \beta_{c2} \int e^{-\alpha \Omega(v(t), r_\tau(k))} dt \tag{10}
\]

In Section IV we will use appropriate weighting to compare physical-only optimization, cyber-only optimization, and total system optimization to demonstrate how increased efficiency and conservation of energy can be achieved by including both physical and cyber objectives.

III. Setup and Solution

Our objective is to survey a straight segment of pipeline by flying a small, high aspect ratio UAS with a downward facing gimbaled camera directly overhead. We have created a simulation in MATLAB to compare various solutions to the optimization problem posed. We first mention the assumptions we’ve made to simplify the problem and demonstrate why an analytical solution is not possible. We then describe the numerical methods chosen to solve our optimization problem, and discuss the models we have adopted.

A. Assumptions

Equation (10) is non-trivial to solve in part due to the need to find the time-varying solution \( v(t) \) and \( r_\tau(k) \). It is further complicated by the discrete nature of the cyber system design variable \( r_\tau(k) \) making this a mixed discrete-continuous equation. We make the following assumptions in order to simplify the problem:

1. The segment of pipeline is straight.
2. We assume aircraft performance is consistent with the principles of steady level flight.
3. The mission takes place close to sea level, with a relatively low altitude allowing use of standard sea level air density.
4. Altitude remains approximately constant through the mission.
5. Due to assumptions 1 and 2, the on-board gimbaled camera always points straight down toward the ground. Specifically, this implies the optical axis of the camera is always perpendicular to the ground plane.
6. We restrict our problem to finding the optimal static \( v \) and \( r_\tau \) that minimize the cost of the mission assuming \( v(t) \) and \( r_\tau(k) \) remain constant throughout.

An interesting addition to this work to be made in the future will be to model certain places on the pipeline as “high interest,” either \( a \text{ priori} \) or through real-time image processing, and therefore solve for the optimal trajectory with dynamically changing velocity and task execution rate.
B. Simplified Cost Function

As a result of the assumptions made we can rewrite Equation (10) as

\[
J(v, r_\tau) = \beta_p \frac{DP(v)}{v_{\text{max}}} + \beta_{p2} \frac{D}{v_{\text{max}}} + \beta_{c1} \frac{r_\tau}{r_{\tau,\text{max}}} + \beta_{c2} \frac{De^{-\alpha \Omega(v, r_\tau)}}{\max \{H\}}.
\]  

(11)

We note that \(r_{\tau,\text{max}}\) is a constant value chosen \textit{a priori} based on the capabilities and demands of the real-time system. \(\max \{E\}\), \(\max \{T\}\) occur at the slowest velocity \(v_{\text{min}}\). \(\max \{H\}\) occurs under conditions giving rise to the lowest amount of overlap between images \((v_{\text{max}}, r_{\tau,\text{min}})\). \(D\) is the total distance of the mission and is constant. We also note that these assumptions allow us to rewrite \(\Omega(v(t), r_\tau(k))\) as

\[
\Omega(v, r_\tau) = 1 - \frac{w v}{r_\tau A}
\]

where \(w_i\) and \(A_i\) denote the constant width and area of an image. In Figure 3 is a plot of \(J(v, r_\tau)\) in Equation (11). We note the convex shape and unconstrained nature of the minimum, implying we should obtain a robust solution with an appropriate numerical optimization method.

After substitutions and some algebra we can rewrite Equation (11) as

\[
J(v, r_\tau) = \beta_p \gamma_1 v^2 + \frac{\beta_{p2} v_{\text{min}}}{v^2} + \beta_{c1} \frac{r_\tau}{r_{\tau,\text{max}}} + \beta_{c2} \gamma_3 e^{\frac{w v}{r_\tau A}}
\]  

(12)

where

\[
\gamma_1 = \frac{v_{\text{min}} C D_0 \rho}{2 P(v_{\text{min}})}
\]

(13a)

\[
\gamma_2 = \frac{2 v_{\text{min}} K W^2}{\rho S P(v_{\text{min}})}
\]

(13b)

\[
\gamma_3 = e^{-\alpha e^{-\alpha \Omega(v_{\text{max}}, r_{\tau,\text{min}})}}.
\]

(13c)

C. Analytical Solution and Feasibility

We investigated the possibility of identifying a minimum for Equation (12) through analytical computation. Due to the constraints of the flight envelope, and of the real-time computing system, let domain \(D \subset \mathbb{R}^2\) be

\[
D = \left\{ \begin{bmatrix} v \\ r_\tau \end{bmatrix} \in \mathbb{R}^2 \left| \begin{bmatrix} v_{\text{min}} \\ r_{\tau,\text{min}} \end{bmatrix} \leq \begin{bmatrix} v \\ r_\tau \end{bmatrix} \leq \begin{bmatrix} v_{\text{max}} \\ r_{\tau,\text{max}} \end{bmatrix} \right. \right\}
\]

(14)

which is a compact set. From the Weierstrass theorem \(J(v, r_\tau)\) in Equation (12) does have a global minimizer.\(^9\)

In attempting to find an analytical solution we formulate the constrained optimization problem

\[
\text{Minimize } J(v, r_\tau) \\
\text{subject to } v \leq v_{\text{max}} \\
v \geq v_{\text{min}} \\
r_\tau \leq r_{\tau,\text{max}} \\
r_\tau \geq r_{\tau,\text{min}}
\]

(15a-e)
However, in applying the Karush-Kuhn-Tucker (KKT) necessary conditions to the Lagrangian we encounter a transcendental function as part of a set of equations with no analytical solution. Using Lagrange variables \( \lambda_i \) for \( i = 1 \ldots 4 \), we have

\[ \nabla_v L = 2\beta_p \gamma_1 v - \frac{2\beta_p \gamma_2}{v^3} - \frac{\beta v_{\min}}{v^2} + \frac{w\beta c \gamma_3}{r \tau A} e^{\frac{w v}{r \tau}} + \lambda_1 - \lambda_2 \]  

\[ \nabla_{r \tau} L = \frac{\beta c_1}{r_{\tau, \max}} - \frac{w v \beta c \gamma_3}{Ar^2} e^{\frac{w v}{r \tau}} + \lambda_3 - \lambda_4 \]  

Even in the case where weights are chosen to eliminate any active constraints, the set of equations in Equation (16) has no analytical solution. This requires us to resort to numerical solutions.

### D. Experimental Models and Setup

1. **Aircraft**

The SolarBubbles student team in the Aerospace Engineering Dept. at the University of Michigan has designed, built, and tested a solar-supplemented powered glider UAS. Their latest platform, SolarSight, has been modeled as part of the design process. In our simulation we presume the aerodynamic and power model parameters given in Table 1. This model leverages the well known power/velocity relationship of a single-engine propeller-driven aircraft as was described in Equation (2). From these model parameters we compute the remaining necessary parameters for the power equation which are shown in Table 2.

**Table 1. UAS Model Parameters**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( e_0 )</td>
<td>Oswald Factor</td>
<td>0.95</td>
</tr>
<tr>
<td>( b )</td>
<td>Wingspan</td>
<td>3.3 m</td>
</tr>
<tr>
<td>( S )</td>
<td>Surface Area</td>
<td>1.127 m²</td>
</tr>
<tr>
<td>( m )</td>
<td>Mass</td>
<td>11.5 kg</td>
</tr>
<tr>
<td>( C_{D_0} )</td>
<td>Zero-lift Drag</td>
<td>0.04</td>
</tr>
</tbody>
</table>

**Table 2. Additional Parameters for Power Equation**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K = \frac{1}{\pi e_0 AR} )</td>
<td>Aerodynamic Parameter</td>
<td>0.0347</td>
</tr>
<tr>
<td>( AR = \frac{b^2}{S} )</td>
<td>Aspect Ratio</td>
<td>9.6628</td>
</tr>
<tr>
<td>( W = gm )</td>
<td>Weight</td>
<td>112.7 kg</td>
</tr>
<tr>
<td>( \rho = \rho_s )</td>
<td>Air Density</td>
<td>1.225 kg/m³</td>
</tr>
</tbody>
</table>

2. **Camera**

**Table 3. Camera Model Parameters**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f )</td>
<td>Focal Length</td>
<td>0.0046 m</td>
</tr>
<tr>
<td>( H_{\text{dist}} )</td>
<td>Horizontal Distance of Image Plane</td>
<td>0.00361 m</td>
</tr>
<tr>
<td>( V_{\text{dist}} )</td>
<td>Vertical Distance of Image Plane</td>
<td>0.00272 m</td>
</tr>
</tbody>
</table>

For the camera we use a standard pin-hole model with parameters taken from a Panasonic GP-CX161/45P/E. The key parameters for this camera are shown in Table 3. Given the pinhole assumption for simplicity, we do not model lens distortions and other effects. Because we know the camera height above ground at all times, presumed constant in steady level flight conditions, we can directly calculate the image footprint on the ground as a function of height above the ground.

3. **Experimental Setup**

For the Solar Sight aircraft, the stall speed (around 9 m/s) and maximum power output of the engine/motor determine the bounds of \( v \). For the cyber rate, \( r_{\tau} \), the lower bound was chosen based primarily on tuning the information cost in Equation 7. That is, at rates lower than 3 Hz there was no overlap between images resulting in coverage gaps thus maximum entropy \((H = 1)\) presumed for the entire mission. The maximum
cyber rate was chosen based on diminishing returns from cyber rates higher than 20 Hz. Bounds are therefore

\begin{align}
9 \text{ m/s} & \leq v \leq 17 \text{ m/s} \\
3 \text{ Hz} & \leq r_\tau \leq 20 \text{ Hz}.
\end{align}

Additionally, via tuning we chose the parameter \( \alpha = 4 \) in Equation (11), and chose the height above ground (from which we are able to derive \( A_i \) and \( w_i \)) to be 30 m.

In our simulation we used MATLAB’s \texttt{fmincon} function to solve the optimization problem. There are a variety of available algorithms, and we obtain equally good results with \texttt{fmincon}’s implementation of the Active-Set and Sequential Quadratic Programming (SQP) algorithms.\(^\text{13}\)

IV. Results

We investigated the impact and tradeoffs between objectives from both the cyber and physical systems with the goal of minimizing energy use and time while maximizing information (minimizing entropy). Our goal is to show that simultaneous consideration of cyber and physical cost terms can yield more capable missions than what would be possible from designing these two parts of the CPS individually. We first examine and analyze Pareto fronts of the cost function in Equation (11) to gain insight into the tradeoffs from competing objectives. We select candidate points along the Pareto front of several plots and use the corresponding \( v \) and \( r_\tau \) to compute associated costs of the mission. We then select weights \( \beta_{p1}, \beta_{p2}, \beta_{c1}, \) and \( \beta_{c2} \), optimize the CPS cost function in Equation (11) and compare results with solution points selected from the Pareto fronts.

A. Pareto Fronts

Pareto front examination and analysis gives insight into the tradeoffs between competing objectives. Pareto front plots of \( J_p \) (Equation (5)) and \( J_c \) (Equation (9)) can be seen in Figure 4 where the black (darker) data points represent the Pareto front.

These curves show how the objectives for the physical and cyber system, individually, trade off their respective costs. The plots in Figure 4 follow their respective governing dynamical equations to produce the curves shown. For Figure 4a the plot is dominated by the power curve indicating we could expend similar amounts of energy, accomplishing our mission in very different lengths of time. Clearly to achieve our minimum time objective, the front side of the power curve is more optimal as indicated by the Pareto front. Because our entropy cost is a function of both \( v \) and \( r_\tau \) we have multiple points corresponding to a
single cyber rate $r_\tau$. As a result, the velocities resulting in a higher entropy cost are dominated by those producing lower entropy.

If we choose a solution along the Pareto front we will optimize for either the physical or cyber portion of the system. Using these plots, we select the velocity corresponding with the data point highlighted in Figure 4a, $v = 14.9 \text{ m/s}$. From the Pareto front for $J_c$, we choose the cyber rate corresponding with the data point highlighted in Figure 4b, or $r_\tau = 5.6 \text{ Hz}$. In Table 4 we show the costs associated with a mission using these parameters.

We can gain more insight into the tradeoffs of the entire cost function by also examining the tradeoffs between the CPS as a whole. We show these Pareto fronts in Figure 5 where in each subfigure we examine the tradeoff between three of the four objectives. In Figure 5a we again observe the presence of the power curve governing the relationship between aircraft energy ($E$) and the other objectives. The curve folds over onto itself and we choose the point indicated in that plot which is in the crease of the function while also balancing cyber utilization ($U_\tau$) and entropy ($H$) costs.

In Figure 5b no new insight or information is gained since the Aircraft Energy cost and Total Time

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$E$</th>
<th>$T$</th>
<th>$U_\tau$</th>
<th>$H$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v = 14.9 \text{ m/s}, r_\tau = 5.6 \text{ Hz}$</td>
<td>16626.2 J</td>
<td>134.2 s</td>
<td>0.28</td>
<td>66.7</td>
</tr>
</tbody>
</table>

Figure 5. Pareto Fronts for $J_c$
cost \((T)\) are independent of Cyber Utilization cost. Additionally, we note the similarity of this plot with the Pareto front for \(J_p\) in Figure 4a. In the Pareto front plot in Figure 5c there are no dominated points making the entire surface a Pareto front. We select the solution point indicated on this Pareto front based on intuition.

Figure 5d shows the tradeoffs between entropy, aircraft energy, and total time costs. In this Pareto front we call attention to the normal tradeoff between total time and aircraft energy costs, but more interestingly the tradeoff with entropy cost. This shows the coupling between cyber and physical cost terms and gives insight into how they compete in the total cost. We follow our previous reasoning in choosing a point that compromises total time and aircraft energy but augmented by an attempt to minimize entropy as well.

We list the velocities, cyber rates, and corresponding costs for each of these three selected points in Table 5.

### Table 5. Parameters and Costs for Data Points Selected from Pareto Fronts

<table>
<thead>
<tr>
<th>Parameters</th>
<th>(E)</th>
<th>(T)</th>
<th>(U_p)</th>
<th>(H)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(v = 14.4 \text{m/s}, r_p = 15.4 \text{Hz})</td>
<td>16314.5 J</td>
<td>138.9 s</td>
<td>0.77</td>
<td>45.2</td>
</tr>
<tr>
<td>(v = 12.4 \text{m/s}, r_p = 6 \text{Hz})</td>
<td>15833.3 J</td>
<td>161.3 s</td>
<td>0.30</td>
<td>58.4</td>
</tr>
<tr>
<td>(v = 12.6 \text{m/s}, r_p = 6.1 \text{Hz})</td>
<td>15816.9 J</td>
<td>158.7 s</td>
<td>0.31</td>
<td>58.4</td>
</tr>
</tbody>
</table>

#### B. Optimization over Total Cost Function \(J(v, r_p)\)

In addition to examining Pareto fronts, using numerical methods, we can solve \(J(v, r_p)\) in Equation (11), examine the resulting costs, and compare them with those found from the Pareto front analysis. This requires we select the weights for each cost term. Often there are auxiliary reasons for favoring one cost term over another such as length of time since the last mission, or a cloudy day with less direct sunshine resulting in a tighter energy budget for our solar powered glider. Since we wish to investigate the comparison of holistic CPS optimization with independent physical and cyber system optimization we allow corresponding weights to go to zero as indicated in the 5th and 6th rows of Table 6. In each case, however, in the absence of any compelling reasons to favor one term over another we equalize all non-zero cost terms as shown. We compare the previous results from our Pareto analysis with our numerical solutions and show the individual costs, as well as the scaled, and normalized total cost in Table 6. The lowest total cost solution is the last entry in the table wherein each individual cost term was given equal weight, and we compare its total cost with the

### Table 6. Comparison of All Solutions

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Solution Type</th>
<th>(E)</th>
<th>(T)</th>
<th>(U_p)</th>
<th>(H)</th>
<th>Total</th>
<th>% Worse than Lowest Cost Soln.</th>
</tr>
</thead>
<tbody>
<tr>
<td>(v = 14.9 \text{m/s}, r_p = 5.6 \text{Hz})</td>
<td>Pareto from Table 4</td>
<td>16626.2 J</td>
<td>134.2 s</td>
<td>0.28</td>
<td>66.7</td>
<td>0.5587</td>
<td>0.25%</td>
</tr>
<tr>
<td>(v = 14.4 \text{m/s}, r_p = 15.4 \text{Hz})</td>
<td>Pareto from Table 5</td>
<td>16314.5 J</td>
<td>138.9 s</td>
<td>0.77</td>
<td>45.2</td>
<td>0.6416</td>
<td>15.13%</td>
</tr>
<tr>
<td>(v = 12.4 \text{m/s}, r_p = 6 \text{Hz})</td>
<td>Pareto from Table 5</td>
<td>15833.3 J</td>
<td>161.3 s</td>
<td>0.30</td>
<td>58.4</td>
<td>0.5682</td>
<td>1.96%</td>
</tr>
<tr>
<td>(v = 12.6 \text{m/s}, r_p = 6.1 \text{Hz})</td>
<td>Pareto from Table 5</td>
<td>15816.9 J</td>
<td>158.7 s</td>
<td>0.31</td>
<td>58.4</td>
<td>0.5663</td>
<td>1.61%</td>
</tr>
<tr>
<td>(v = 15.2 \text{m/s}, r_p = 18 \text{Hz})</td>
<td>(J) with (\beta_{p1} = \beta_{p2} = 0.5, \beta_{c1} = \beta_{c2} = 0.0)</td>
<td>16844.1 J</td>
<td>131.6 s</td>
<td>0.90</td>
<td>44.3</td>
<td>0.6708</td>
<td>20.37%</td>
</tr>
<tr>
<td>(v = 9.0 \text{m/s}, r_p = 4.3 \text{Hz})</td>
<td>(J) with (\beta_{p1} = \beta_{p2} = 0.0, \beta_{c1} = \beta_{c2} = 0.5)</td>
<td>19722.8 J</td>
<td>222.2 s</td>
<td>0.22</td>
<td>58.7</td>
<td>0.6654</td>
<td>19.40%</td>
</tr>
<tr>
<td>(v = 14.2 \text{m/s}, r_p = 5.6 \text{Hz})</td>
<td>(J) with (\beta_{p1} = \beta_{p2} = 0.25, \beta_{c1} = \beta_{c2} = 0.25)</td>
<td>16208.8 J</td>
<td>140.8 s</td>
<td>0.28</td>
<td>64.9</td>
<td>0.5573</td>
<td>N/A</td>
</tr>
</tbody>
</table>
other solutions as a percentage.

V. Conclusions and Future Work

As technology allows us to shrink physical platform size, resource use by a cyber system (e.g., the computational and communication components) begins to rival actuation effort of the physical system required for propulsion and servo actuation. This paper investigates holistic optimization over both, demonstrating by example that co-design of the cyber-physical system results in a net savings of energy for given mission time and information gain by efficiently allocating CPS resources.

We have demonstrated such a coupled co-design in the form of optimizing over cost functions describing competing cyber and physical objectives. A UAS surveillance mission of a straight segment of pipeline was proposed and simulation results were obtained. We analyzed Pareto fronts of these results showing important tradeoffs between aircraft airspeed and task execution rate of an important mission real-time system task. We found optimal solutions to our combined CPS cost function and compared those results with independently optimized cyber and physical cost functions, demonstrating that large efficiency improvements can be realized by such an approach.

An important improvement on this work will be to demonstrate a dynamic real-time planner that can appropriately adjust parameters based on feedback from mission objectives, which in our example translates to feedback from imagery about the state of the pipeline. Additionally, this work will ultimately be combined with our previous work in which we balance mission critical cyber tasks with physical system performance leading to a well-rounded co-design process for CPS.

References